A Novel Higher Order Sliding Mode Control: Application to an Induction Motor

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Abstract — **In this paper, we propose a novel higher order sliding mode control used to solve the problem of chattering phenomenon related to the standard sliding mode controller. The proposed controller allows obtaining an exponential stability as well as a finite time convergence to the sliding surface and guarantees the robustness of the closed loop system against uncertainties and external matched disturbances. This algorithm is applied to control the speed, the angular position and the rotor flux of an induction motor. Numerical simulations are developed to show the efficiency and to evaluate the robustness of the proposed controller against external disturbances.**

Keywords — Nonlinear systems, robustness, higher order sliding mode control, asynchronous machine.

I. INTRODUCTION

The sliding mode control (SMC) has amply demonstrated its effectiveness through theoretical and practical studies. Its main areas of application include robotics and electrical machinery [4], [6]. Such control technique is well known by its robustness against external matched disturbances, parametric variations, modeling uncertainties and nonlinearities (hysteresis, friction, etc.) that often characterize dynamical systems. Indeed, SMC is able to overcome these barriers in regulation and tracking. The robustness property is achieved by using a high frequency switching to steer the states of a system into the sliding surface [16].

The high-frequency switching leads, generally, to the appearing of an undesirable chattering of the control input. As a result, a large energy is lost in electric actuators leading to a rapid wear of mechanical actuators [4]. To overcome this problem, several solutions have been proposed in the literature. Among them, we replace the "*sign*" function by the saturation function or the sigmoid function in order to obtain a smoother control signal [4]. Besides, fuzzy logic may be used together with SMC [14].

The most interesting way to get rid of the chattering phenomenon consists of enforcing a higher-order sliding mode (HOSM). The main objective of SMC of order ρ

(called ρ-SMC) is to obtain a finite time convergence onto the manifold $S^{\rho} = \left\{ s = \dot{s} = \ldots = s^{(\rho-1)} = 0 \right\}$, s is the sliding variable. So, the control acts on s and its higher derivatives to force the sliding variable and its ρ-1 first time derivatives to zero in finite time [9].

In [10], authors proposed HOSMC that does not depend on the dynamics of the system and which guarantees the robustness of the closed loop system. Laghrouche et al. [8] developed a controller based on the minimization of a quadratic criterion using the concept of sliding mode control with integral action. This allows stabilizing in finite time a system of high order on the sliding surface. Besides, it permits to choose in advance the convergence time to the sliding surface. Although these algorithms are general, a priori accurate knowledge of the initial conditions of the system limit seriously the applicability of these approaches.

Mondal et al. [13] proposed a second order sliding mode controller based on a nonlinear sliding surface to control uncertain linear systems with matched uncertainty. The stability of the nonlinear sliding manifold is guaranteed and the chattering of the control input is reduced.

Recently, Defoort et al. [2]-[5] proposed a finite time HOSMC for a class of multivariable nonlinear systems. In such work, authors remove the drawbacks devoted in [8] and presented a method used to stabilize in finite time the higher order input output dynamic system with bounded uncertainties. The most advantage of this control strategy consists to obtain smooth states trajectories of the system around the sliding surface and this control technique can give a simple relationship between the synthesis parameters of the control and the desired performances desired in closed loop. However, this technique of higher-order sliding mode control contains necessarily a discontinuous part to reject the effect of disturbances. Its design requires the knowledge of the maximum amplitudes of the disturbances acting on the system. This approach ensures an asymptotic stability.

In this paper, we presented a new technique of HOSMC, and we demonstrated its exponential stability and robustness against external matched disturbances, parametric variations and modeling uncertainties.

The induction motors (IM) is widely used in industry, this is mainly due to its rigidness. Also, its maintenance is free operation, and relatively low cost. Moreover, induction motors constitute a theoretically challenging control problem since the dynamical system is nonlinear. The technique of vector control by indirect field oriented applied to induction motors permitted to have better performances comparable to a DC motor. Nevertheless, it is very sensitive to parametric variations and external disturbances. To solve this problem, we present in this paper an application of the novel robust higher order sliding mode control to the model of IM. Simulations results developed in this work show the effectiveness of the proposed HOSMC and the simplicity of adjustment of the synthesis parameters of the control law to achieve the desired performances in closed loop.

This paper is organized as follows. The next section is devoted to the problem formulation. In section 3, we proposed the new HOSMC and we focus on the main contribution of the paper. In section 4, we presented a model of induction motor. The simulations results are given in section 5. Conclusions are reported in the last section of the paper.

II. PROBLEM FORMULATION

Consider a nonlinear dynamical system described by the following equation

$$\dot{x} = f(x,t) + g(x,t)u + p(t)$$
$$s = s(x,t)$$
(1)

where $x = [x_1, ..., x_n]^T \in X$ is the state variable of the system with X an open set of $\Re^n$ and $u \in U$ the control input is a feature possibly discontinuous and bounded, depending on time and the system state, with U is an open set $\Re$, $f(x,t)$ and $g(x,t)$ are sufficiently differentiable vector fields and $p(t) \in \Re^n$ is an additive perturbation.

Assumption 1. System (1) admits a $\rho \in N$ constant and known relative degree with respect to the sliding variable $s(x,t)$.

The system (1) can be written as follows:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{\rho-1} &= z_\rho \\ \dot{z}_\rho &= \phi(x,t) + \varphi(x,t)u \end{cases}$$
(2)

with

$$\begin{cases} \phi &= \bar{\phi} + \delta_\phi \\ \varphi &= \bar{\varphi} + \delta_\varphi \end{cases}$$

where $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_\rho \end{bmatrix}^T = \begin{bmatrix} s & \dot{s} & \cdots & s^{(\rho-1)} \end{bmatrix}^T$,

$\bar{\phi}$ and $\bar{\varphi}$ are nominal known parts, δ_φ and δ_ϕ are unknown parts, including disturbances and uncertainties.

Assumption 2. The nominal part $\bar{\varphi}$ is assumed invertible.

The objective of the different techniques of higher order sliding mode control is to obtain a finite time convergence onto the manifold $S^\rho = \{s = \dot{s} = ... = s^{(\rho-1)} = 0\}$. To achieve such goal, we propose a new technique of HOSMC.

III. HIGHER ORDER SLIDING MODE CONTROLLER

Consider a chain of integrators, defined by

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{\rho-1} &= z_\rho \\ \dot{z}_\rho &= z_{\rho+1} = w(z) \end{cases}$$
(3)

To stabilize the system (3) in finite time on S^ρ, we propose the following control law.

$$w(z) = -\alpha \frac{(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho)}{(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|)}$$
(4)

with

$\alpha > 0$, $a_i > 0$ $(1 \le i \le \rho)$, $b_i > 0$ $(1 \le i \le \rho)$ and the polynomial $P(x) = a_1 + a_2 x + \cdots + a_\rho x^{\rho-1}$ is Hurwitz.

Now consider the control law

$$u = \bar{\varphi}^{-1}(w(z) - \bar{\phi})$$
(5)

where $\bar{\phi}$ and $\bar{\varphi}$ are obtained according to (2) and $w(z)$ is given by (4).

Theorem : The controller (5) ensures a sliding mode of order ρ with respect to $s(x,t)$ provided that assumptions (2) and (3) are verified.

Proof.

Equation (4) can be written as follows

$$(a_1 s + a_2 \dot{s} + \cdots + a_\rho s^{(\rho-1)}) = f(t)$$
(6)

with

$$f(t) = -\dot{z}_\rho \frac{(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|)}{\alpha}$$
(7)

The Laplace Transform applied to (6) gives

$$S(p)(a_1 + a_2 p + \cdots + a_\rho p^{\rho-1}) = F(p)$$
(8)

As $P(p) = a_1 + a_2 p + \cdots + a_\rho p^{\rho-1}$ is Hurwitz, the solution of (6) is stable.

proof of the convergence of $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_\rho \end{bmatrix}^T$ to

the zero vector of $\Re^{\rho}$.

Assume that the states of system are not in the manifold s^{ρ}. One has

$$z_{\rho+1} = w(z) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \quad (9)$$

In other words, one obtains

$$\beta z_{\rho+1} = a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho \quad (10)$$

with

$$\beta = -\frac{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)}{\alpha} < 0 \; ; \forall t$$

So, (10) is assumed a linear differential equation with second member. The equation without second member is given by

$$a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho = 0 \quad (11)$$

The roots of the polynomial $P(p)$ have a strictly negative real part. So, the polynomial $P(p)$ which is given by

$$P(p) = a_1 + a_2 p + \cdots + a_\rho p^{\rho-1} \quad (12)$$

can be rewritten as follows

$$P(p) = \prod_{i=1}^{r} (p - \lambda_i)^{\eta_i} \;\; , \;\; \sum_{i=1}^{r} \eta_i = \rho - 1 \;\; , \;\; \mathrm{Rel}(\lambda_i) < 0$$

with λ_i are the roots of the polynomial $P(p)$ with multiplicity degree η_i.

Therefore, the homogeneous solution of (10) is of the form

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t} \quad (13)$$

with $q_i(t)$ are polynomials of degree $\eta_i - 1$.

Also, one notes that the null function $z_1 = 0$ is a solution of the equation (10). Indeed if $z_1 = 0 \; ; \forall t$, $\beta = 0 \; ; \forall t$. So, one can take it as a particular solution of (10). Consequently, the general solution of (10) is given by

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t} \quad (14)$$

The coefficients of $q_i(t)$ are fixed using initial conditions. So the $z_i = z_1^{(i-1)}$ $(1 \le i \le \rho)$ converge exponentially to zero of $\Re^{\rho}$ whatever the initial conditions are chosen.

Proof of robustness

Suppose that the control is affected by the disturbances as follows

$$p_1(t) z_{\rho+1} + p_2(t) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \quad (15)$$

where $z_{\rho+1} = \dot{z}_\rho$, $p_1(t)$ and $p_2(t)$ are two bounded disturbances.

Equation (15) can be rewritten as follows

$$\beta \left(p_1(t) z_{\rho+1} + p_2(t)\right) = \left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right) \quad (16)$$

The homogeneous solution of the equation (16) takes the form

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t}$$

and $z_1 = 0$ is a particular solution of (16). So the general solution of (16) is given by

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t}$$

So, $z_i = z_1^{(i-1)}$ $(1 \le i \le \rho)$ converge exponentially to zero of $\Re^{\rho}$. Therefore, the proposed controller ensures the robustness against bounded disturbances.

Now, consider system (2) which can be rewritten as follows:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_{\rho-1} &= z_\rho \\ \dot{z}_\rho &= v(x,t) + (1 + \varsigma(x,t)) w(z) \end{cases} \quad (17)$$

with

$$u = \overline{\varphi}^{-1} \left(w - \overline{\phi}\right) \quad \text{and} \quad \begin{cases} v &= \delta_\phi - \delta_\varphi \overline{\varphi}^{-1} \overline{\phi} \\ \varsigma &= \delta_\varphi \overline{\varphi}^{-1} \end{cases}$$

The functions $v(x,t)$ and $\varsigma(x,t)$ may include the uncertainties of the system.

Assumption 3. The functions $v(x,t)$ and $\varsigma(x,t)$ are bounded. In addition, there is a positive function $a(x)$ and b a positive constant $0 < b \le 1$, such that:

$$\begin{cases} |v(x,t)| &\le a(x) \\ |\varsigma(x,t)| &\le 1 - b \end{cases} \quad (18)$$

Using (15), one has

$$\dot{z}_\rho = v(x,t) + (1 + \varsigma(x,t)) w(z) \quad (19)$$

If Assumption (3) is verified, one can write (19) as follows

$$\begin{aligned} w(z) &= (1 + \varsigma(x,t))^{-1} (\dot{z}_\rho - v(x,t)) \\ &= (1 + \varsigma(x,t))^{-1} \dot{z}_\rho - (1 + \varsigma(x,t))^{-1} v(x,t) \end{aligned} \quad (20)$$

Now, applying our approach (15) to (20) one obtains

$$p_1(t) z_{\rho+1} + p_2(t) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \quad (21)$$

with

$$p_1(t) = \left(1 + \varsigma(x,t)\right)^{-1} \tag{22}$$

$$p_2(t) = -\left(1 + \varsigma(x,t)\right)^{-1} v(x,t) \tag{23}$$

Now, using the proof of robustness presented above, one can conclude that the control law (4) allows to stabilize exponentially the uncertain system (1) on the sliding surface in finite time.

Remark : To implement the controller (4), a finite time differentiator is used to estimate the successive derivatives $\left(\dot{s}, \ddot{s}, \cdots, s^{(\rho-1)}\right)$ of the sliding variable (s) [11].

IV. MATHEMATICAL MODEL OF THE INDUCTION MOTOR

The modeling of the induction machine described in the repository Park is given in the following system of equations [1], [7], [12], [15].

$$\begin{cases} \dfrac{di_{sd}}{dt} = -\dfrac{L_{sr}^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_{sd} + \dfrac{L_{sr} R_r}{\sigma L_s L_r^2} \phi_{rd} + \omega_a i_{sq} + \dfrac{1}{\sigma L_s} u_{sd} \\[3mm] \dfrac{di_{sq}}{dt} = -\dfrac{L_{sr}^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_{sq} - \dfrac{pL_{sr}}{\sigma L_s L_r} \omega \phi_{rd} - \omega_a i_{sd} + \dfrac{1}{\sigma L_s} u_{sq} \\[3mm] \dfrac{d\phi_{rd}}{dt} = -\dfrac{R_r}{L_r} \phi_{rd} + \dfrac{L_{sr} R_r}{L_r} i_{sd} \\[3mm] \dfrac{d\omega}{dt} = \dfrac{C_{em}}{J} - \dfrac{f}{J} \omega - \dfrac{C_r}{J} \\[3mm] \dfrac{d\theta}{dt} = \omega \end{cases} \tag{24}$$

where R_r and R_s are rotor and stator resistances, L_r and L_s are rotor and stator inductances, L_{sr} is the mutual inductance, θ is the rotor position, ω is the rotor angular velocity, p is the number of pole pairs, J is the inertia of the rotor, f is the coefficient of viscous friction, C_r is the load torque, ϕ_{rd} is the rotor flux linkage, i_{sd} and i_{sq} stand for the d-q axis currents, u_{sd} and u_{sq} are the d-q axis voltages, $\sigma = 1 - \dfrac{L_{sr}^2}{L_s L_r}$, is the dispersion coefficient of Blondel, $C_{em} = \dfrac{pL_{sr}}{L_r} \phi_{rd} i_{sd}$ is the electromagnetic torque and $\omega_a = \left(p\omega + \dfrac{L_{sr} R_r}{L_r \phi_{rd}} i_{sq}\right)$.

One can remark that the model of the IM (24) can be written in the form (1) with

$$\dot{x} = f(x,t) + g(x,t)u + p(t)$$

with

$$x = \begin{bmatrix} i_{sd} & i_{sq} & \phi_{rd} & \omega \end{bmatrix}^T$$

$$f(x,t) = \begin{bmatrix} -\dfrac{L_{sr}^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_{sd} + \omega_a i_{sq} + \dfrac{L_{sr} R_r}{\sigma L_s L_r^2} \phi_{rd} \\[4mm] -\omega_a i_{sd} - \dfrac{L_{sr}^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} i_{sq} - \dfrac{pL_{sr}}{\sigma L_s L_r} \omega \phi_{rd} \\[4mm] \dfrac{L_{sr} R_r}{L_r} i_{sd} - \dfrac{R_r}{L_r} \phi_{rd} \\[4mm] \dfrac{pL_{sr}}{L_r J} \phi_{rd} i_{sd} - \dfrac{f}{J} \omega \end{bmatrix}$$

$$g(x,t) = \begin{bmatrix} \dfrac{1}{\sigma L_s} & 0 & 0 & 0 \\[3mm] 0 & \dfrac{1}{\sigma L_s} & 0 & 0 \end{bmatrix}^T, \quad u = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix}$$

$$p(t) = \begin{bmatrix} 0 & 0 & 0 & -\dfrac{C_r}{J} \end{bmatrix}^T$$

V. APPLICATION TO THE ASYNCHRONOUS MACHINE

A. Control of Speed and Rotor flux

In this section, the control objective is to apply an output feedback control scheme ensuring a higher order sliding mode in order to enforce the rotor angular velocity ω and the rotor flux linkage ϕ_{rd}, to track a desired trajectory ω_{ref} and $\phi_{rd\,ref}$. So, we consider the following sliding variable

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \omega - \omega_{ref} \\ \phi_{rd} - \phi_{rd\,ref} \end{bmatrix} \tag{25}$$

As the relative degree of the system with respect s is $\rho = 2$ a 2-SMC can be designed. So, the second time derivative of the sliding variable is given by:

$$\ddot{s} = \left(\phi(x,t) + \varphi(x,t)u\right)$$

where

$$\overline{\phi}(x,t) = \begin{bmatrix} \overline{\phi}_1(x,t) \\ \overline{\phi}_2(x,t) \end{bmatrix}$$

$$\overline{\phi}(x,t) = \begin{bmatrix} \alpha_1 i_{sd} + \alpha_2 i_{sq} + \alpha_3 \phi_{rd} + \alpha_4 \omega - \ddot{\omega}_{ref} \\ \beta_{12} i_{sd} i_{sq} + \beta_{13} i_{sd} \phi_{rd} + \beta_{23} i_{sq} \phi_{rd} + \beta_{334} \phi_{rd}^2 \omega + \beta_4 - \ddot{\phi}_{rd\,ref} \end{bmatrix} \tag{26}$$

with

$$\begin{cases} \alpha_1 = -\left(\dfrac{R_r^2 L_{sr}}{L_r^2} + \dfrac{L_r^3 R_r^2 + L_{sr} L_r^2 R_s R_r}{\sigma L_s L_r^3} \right) \\[4mm] \alpha_2 = \left(\dfrac{L_{sr} R_r}{L_r} \omega_a \right) \\[4mm] \alpha_3 = \left(\dfrac{L_{sr}^2 R_r^2}{\sigma L_s L_r^3} \right) \\[4mm] \alpha_4 = 0 \end{cases}$$

and

$$\begin{cases} \beta_{12} = \dfrac{pL_{sr}^2 R_r}{L_r^2 J} \\[4mm] \beta_{13} = -\dfrac{pL_{sr}}{L_r J} \omega_a \\[4mm] \beta_{23} = -\left(\dfrac{pL_{sr} R_r}{L_r^2 J} + \dfrac{pL_{sr}}{L_r J}\left(1 + \dfrac{L_{sr}^2 R_r + L_r^2 R_s}{\sigma L_s L_r^2} \right) \right) \\[4mm] \beta_{334} = -\dfrac{p^2 L_{sr}^2}{\sigma L_s L_r^3 J} \\[4mm] \beta_4 = -\dfrac{f}{J} \end{cases}$$

where

$$\overline{\varphi}(x,t) = \begin{bmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{bmatrix} \text{ with } \lambda_1 = \dfrac{pL_{sr}}{\sigma L_s L_r J}\phi_{rd} \; ; \; \lambda_2 = \dfrac{L_{sr} R_r}{\sigma L_s L_r}$$

$$\delta_\varphi = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \delta_\phi = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applied to the model of the induction motor, the higher order sliding mode control (4) is given by:

$$u = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} = \overline{\varphi}^{-1} \left[w(z) - \overline{\phi} \right]$$

with

$$w(z) = \begin{bmatrix} w_1(z_1) \\ w_2(z_2) \end{bmatrix} = \begin{bmatrix} -\alpha_1 \dfrac{(a_{11} z_{1,1} + a_{21} z_{2,1})}{(b_{11}|z_{1,1}| + b_{21}|z_{2,1}|)} \\[5mm] -\alpha_2 \dfrac{(a_{12} z_{1,2} + a_{22} z_{2,2})}{(b_{12}|z_{1,2}| + b_{22}|z_{2,2}|)} \end{bmatrix}$$

$$w_1(z_1) = -2 \cdot 10^5 \cdot \dfrac{(800 \cdot z_{1,1} + 10^{-3} \cdot z_{2,1})}{(|z_{1,1}| + |z_{2,1}|)}$$

obtained for $\alpha_1 = 2 \cdot 10^5$, $a_{11} = 800$, $a_{21} = 10^{-3}$, $b_{11} = 1$ and $b_{21} = 1$.

$$w_2(z_2) = -10^4 \cdot \dfrac{(2,15 \cdot z_{1,2} + z_{2,2})}{(10 \cdot |z_{1,2}| + |z_{2,2}|)}$$

obtained for $\alpha_2 = 10^4$, $a_{12} = 2,15$, $a_{22} = 1$, $b_{12} = 10$ and $b_{22} = 1$.

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} [z_{1,1} \;\; z_{2,1}]^T \\ [z_{1,2} \;\; z_{2,2}]^T \end{bmatrix} = \begin{bmatrix} [s_1 \;\; \dot{s}_1]^T \\ [s_2 \;\; \dot{s}_2]^T \end{bmatrix}$$

For this application, we assume $\ddot{\omega}_{ref} = 0$ and $\dddot{\phi}_{rd\,ref} = 0$

First, we consider the case where noises are absents. So the load torque $C_r(t) = 0$.

Simulation results plotted on figures (1)-(3), show that control objective is fulfilled using this algorithm.
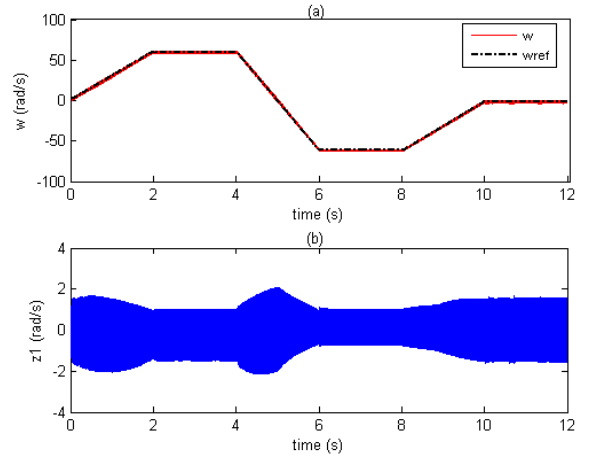

Fig. 1. Tracking performances without noises, (a) ω and ω_{ref}, (b) $z_1 = s_1 = \omega - \omega_{ref}$.
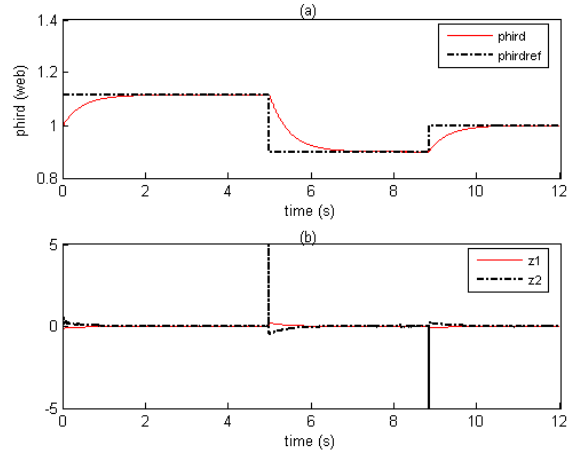

Fig. 2. Tracking performances without noises, (a) ϕ_{rd} and $\phi_{rd\,ref}$, (b) sliding surface $z_1 = s_2$ and $z_2 = \dot{s}_2$.
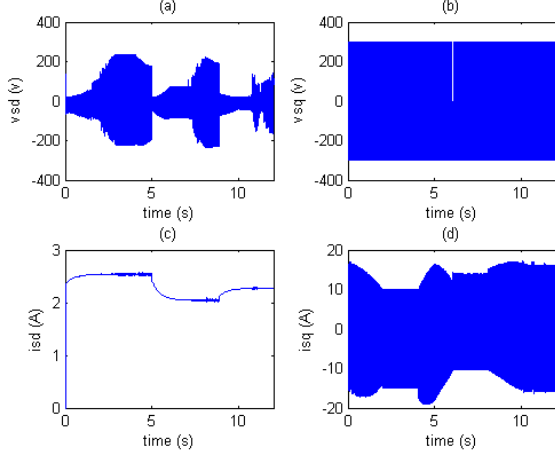
Fig. 3. Controls signals and the stator currents without noises

Now, we consider that our system is affected by external disturbance. So, the load torque varies randomly as plotted on figure 4. Simulations results depicted on figures (5)-(7) show the robustness of this technique of control against external disturbances.
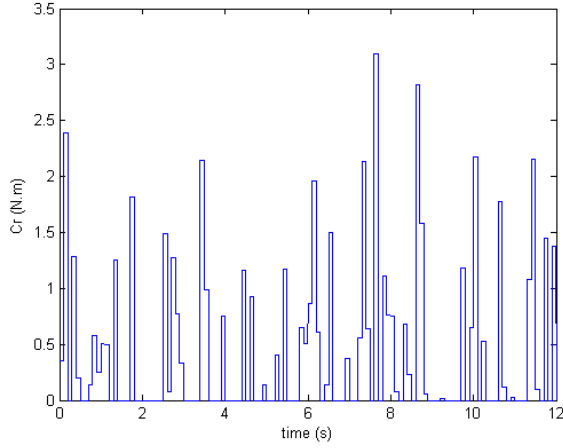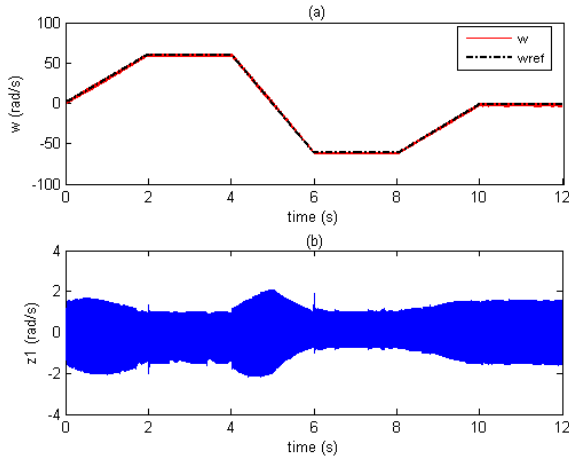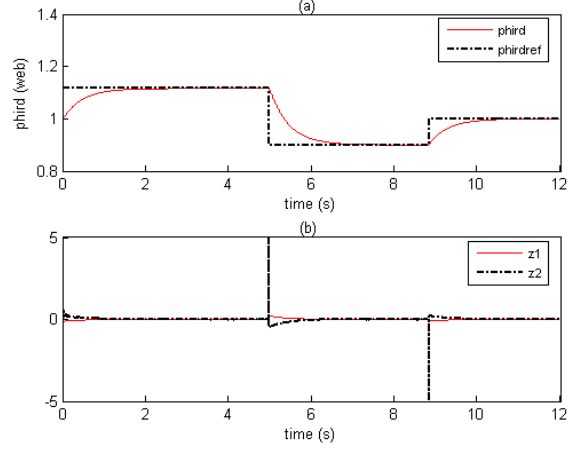

Fig. 4. Evolution of the Load torque C_r .

Fig. 5. Tracking performances under disturbances, (a) ω and ω_{ref} , (b)

$$z_1 = s_1 = \omega - \omega_{ref} .$$

Fig. 6. Tracking performances under disturbances, (a) ϕ_{rd} and $\phi_{rd\,ref}$,

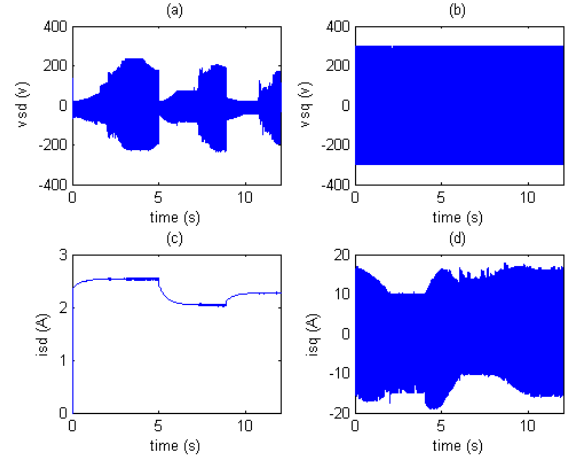(b) sliding surface $z_1 = s_2$ and $z_2 = \dot{s}_2$

Fig. 7. Controls signals and the stator currents under disturbances.

B. Control of the position and the rotor flux

In this part, the objective is to design a 3rd order SMC to enforce the rotor angular position θ to track a desired trajectory θ_{ref} and to develop 2nd order SMC to drive the rotor flux linkage ϕ_{rd} to track a desired trajectory $\phi_{rd\,ref}$. So, we consider the following sliding variable

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \theta - \theta_{ref} \\ \phi_{rd} - \phi_{rd\,ref} \end{bmatrix} \tag{27}$$

$$\begin{bmatrix} \dddot{s}_1 \\ \ddot{s}_2 \end{bmatrix} = \left(\phi\left(x,t\right) + \varphi\left(x,t\right) u \right)$$

So according to the expression of the matrix $\overline{\varphi}\left(x,t\right)$, the input control $u_1 = u_{sd}$ depend only on s_2 and its first derivatives. Besides, $u_2 = u_{sq}$ depend only on s_2, $\dot{s}_2$ and $\ddot{s}_2$, Consequently, the control law applied to the IM is given by.

$$u = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} = \overline{\varphi}^{-1} \left[w\left(z\right) - \overline{\phi} \right]$$

with

$$w\left(z\right) = \begin{bmatrix} w_1\left(z_1\right) \\ w_2\left(z_2\right) \end{bmatrix} = \begin{bmatrix} -\alpha_1 \dfrac{\left(a_{11}z_{1,1} + a_{21}z_{2,1} + a_{31}z_{3,1}\right)}{\left(b_{11}\left|z_{1,1}\right| + b_{21}\left|z_{2,1}\right| + b_{31}\left|z_{3,1}\right|\right)} \\ -\alpha_2 \dfrac{\left(a_{12}z_{1,2} + a_{22}z_{2,2}\right)}{\left(b_{12}\left|z_{1,2}\right| + b_{22}\left|z_{2,2}\right|\right)} \end{bmatrix}$$

where

$$w_1\left(z_1\right) = -9,5\cdot10^4 \cdot \frac{\left(20\cdot z_{1,1} + 2000\cdot z_{2,1} + 0,5\cdot z_{3,1}\right)}{\left(3000\cdot\left|z_{1,1}\right| + 48,5\cdot\left|z_{2,1}\right| + \left|z_{3,1}\right|\right)}$$

obtained for $\alpha_1 = 9,5\cdot10^4$, $a_{11} = 20$, $a_{21} = 2000$, $a_{31} = 0,5$, $b_{11} = 3000$, $b_{21} = 48,5$ and $b_{31} = 1$.
and

$$w_2\left(z_2\right) = -10^4 \cdot \frac{\left(2,15\cdot z_{1,2} + z_{2,2}\right)}{\left(10\cdot\left|z_{1,2}\right| + \left|z_{2,2}\right|\right)}$$

obtained for $\alpha_2 = 10^4$, $a_{12} = 2,15$, $a_{22} = 1$, $b_{12} = 10$ and $b_{22} = 1$.
where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} z_{1,1} & z_{2,1} & z_{3,1} \end{bmatrix}^T \\ \begin{bmatrix} z_{1,2} & z_{2,2} \end{bmatrix}^T \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} s_1 & \dot{s}_1 & \ddot{s}_1 \end{bmatrix}^T \\ \begin{bmatrix} s_2 & \dot{s}_2 \end{bmatrix}^T \end{bmatrix}$$

First, we consider the load torque $C_r\left(t\right) = 0$. Simulation results plotted on figures (8)-(10) show that the control objective is fulfilled using the proposed HOSMC. Besides, we note on figure 9 that the new controller of the rotor angular position θ minimizes considerably the chattering phenomenon on speed error $z_{2,1}$. Moreover, we show on figure 11 that the rotor angular position controller improve the control signal applied to the system.
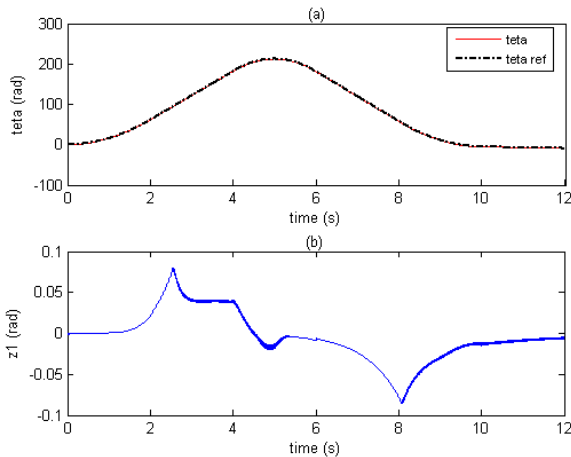

Fig. 8. Tracking performances without noises, (a) θ and θ_{ref}, (b)
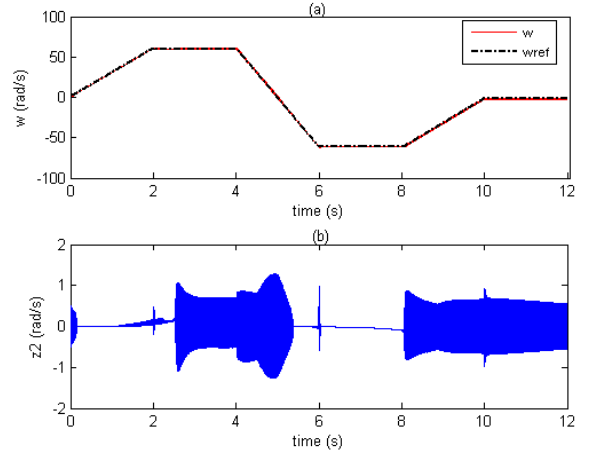
$$z_1 = s_1 = \theta - \theta_{ref}.$$

Fig. 9. Tracking performances without noises, (a) ω and ω_{ref}, (b)
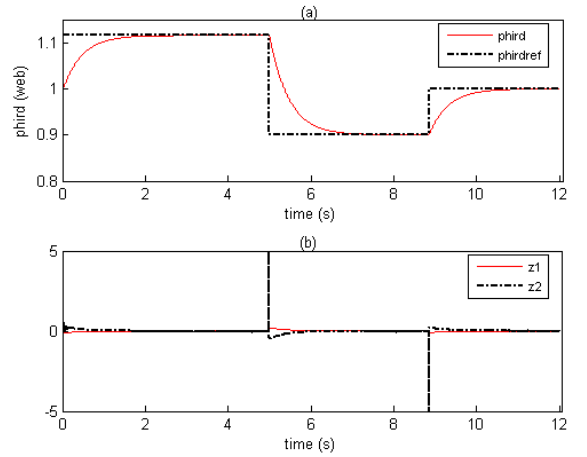
$$z_2 = \dot{s}_1 = \omega - \omega_{ref}.$$

Fig. 10. Tracking performances without noises, (a) ϕ_{rd} and $\phi_{rd\,ref}$, (b)

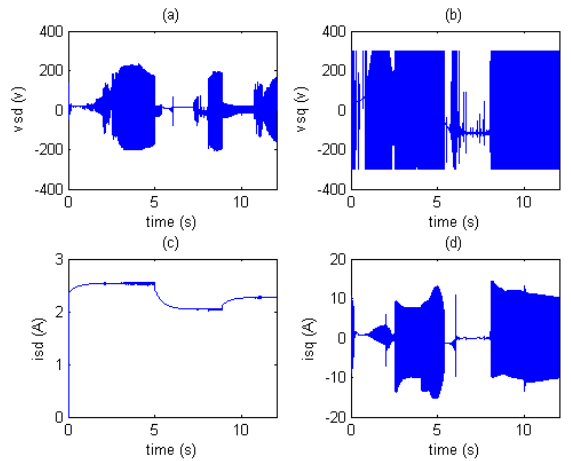sliding surface $z_1 = s_2$ and $z_2 = \dot{s}_2$.

Fig. 11. Controls signals and the stator currents without noises

Now, we consider that the IM is affected by an external disturbance. So, the load torque varies randomly and it is presented on figure 4.

Simulations results depicted on figures (12)-(15) show the

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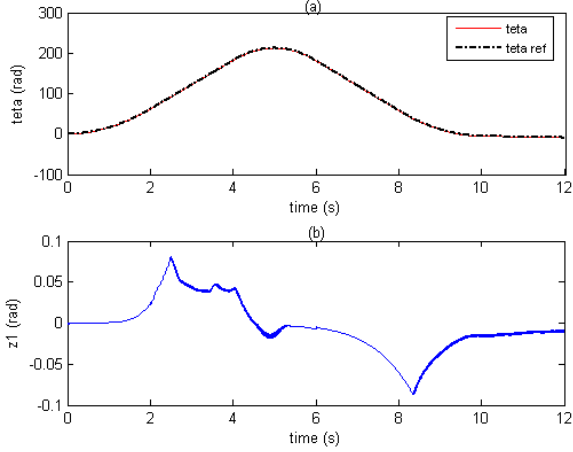robustness of the proposed approach against external disturbances.

Fig. 12. Tracking performances under disturbances, (a) θ and θ_{ref}, (b)
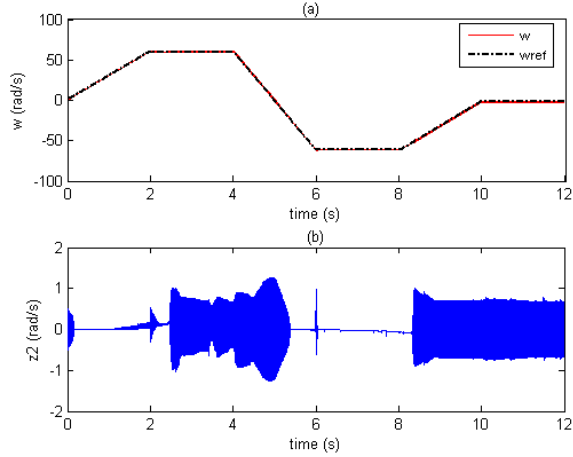
$$z_1 = s_1 = \theta - \theta_{ref}.$$

Fig. 13. Tracking performances under disturbances, (a) ω and ω_{ref}, (b)

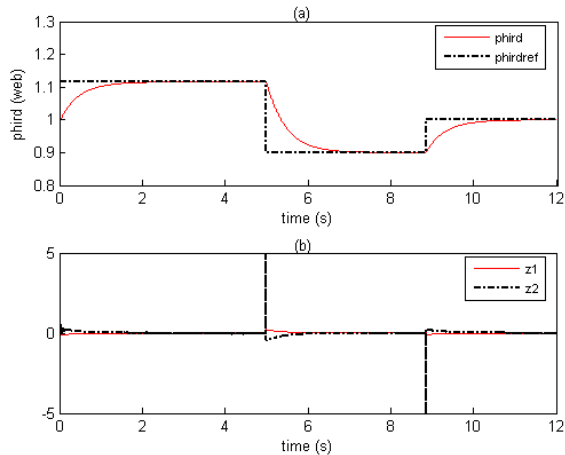$$z_2 = \dot{s}_1 = \omega - \omega_{ref}.$$

Fig. 14 Tracking performances under disturbances, (a) ϕ_{rd} and $\phi_{rd\ ref}$,

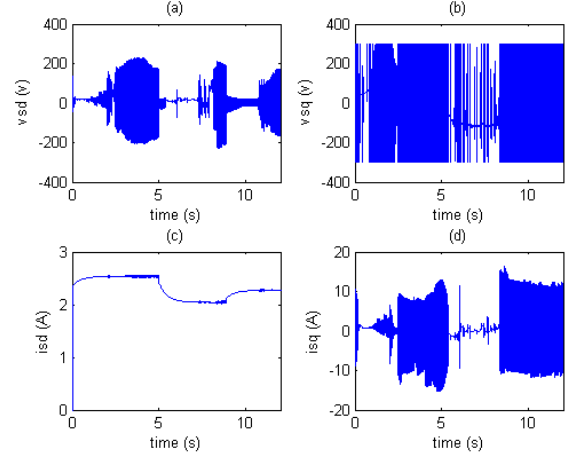(b) sliding surface $z_1 = s_2$ and $z_2 = \dot{s}_2$.

Fig. 15. Controls signals and the stator currents under disturbances.

VI. Conclusion

In this work, we have proposed a novel robust high-order sliding mode controller for a class of nonlinear uncertain systems. The proposed controller provides for an exponential stability of the closed loop system. Furthermore, we have applied this approach to the model of an induction motor. Simulation results show the high performances and the robustness of such control strategy against external matched disturbances.

Nomenclature

Table 1: Characteristics and parameters of induction motor

Nominal puissance	1.5 KW	Rs	8
Voltage	220/380 V	Rr	4
Nominal current	6.4/3.7 A	Ls	0.47
Number of pole pair	P=2	Lr	0.47
Frequency	50 Hz	Lsr	0.44
rated speed	1410 tr/min	J Kg m^2	J=0.04
Factor of puissance	0.83	f N msec/rad	f=0.002

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